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Article

Effects of variable amplitudes of corrugated surface on the dispersion of surface wave on a heterogeneous impedance half-space

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Abstract: A Mathematical model analyzing the dispersion of surface waves through the exploration of the theory of fibre-reinforced half-space, corrugation and heterogeneous impedance conditions at the boundary is investigated. Using the constitutive equations of fibre-reinforced medium and its governing relations, the equations of motion were derived. Closed-form solutions of stresses and displacements of the wave on the material are presented by employing the eigenvalue approach otherwise called the normal mode method. Following this, the analytical and graphical solutions of the results are presented by employing the inhomogeneous impedance and variable corrugation effects at the boundary of the material. We depicted the corrugation effects in a way that the amplitudes of corrugation are dependent on the horizontal coordinate of space which leads to variable amplitudes at the boundary. Impact of the quantities of wavenumber, variable amplitudes of corrugation, and heterogeneous parameters demonstrates various degree of exhibitions on the dispersion of the Rayleigh wave. One of the parameters associated with variable amplitudes of corrugation cause a downward trend to the dispersion of the Rayleigh wave when its value increases on the solid while its counterpart demonstrate a very clear increase in behavior on the dispersions of the Rayleigh wave in certain domains of the horizontal length of the material. The inhomogeneous parameter decreases the dispersion of the Rayleigh wave when its value increase. Thus, we emphasize that this investigation would aid research analysis in geophysics, analysis on surfaces linked to seismology, material designs cum manufacturing, etc.

Keywords: variable amplitudes of corrugation; inhomogeneous impedance boundaries; Rayleigh wave; inhomogeneous fibre-reinforced solid; wavenumber.

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1. Introduction

Material behavior and wave phenomena are some concepts that underpins the study of solid mechanics of materials. In classical theory of elasticity and wave propagation on these materials, for instance, local relationship between stress and strain on material bodies during deformations are in contrast when nonlocal theory of elasticity are incorporated into the model problem. This is because nonlocal theories prescribes or account for the fact that the stress at a point in a medium is dependent on the given strain at that point and also the strain at the neighborhood of points. In fact, mathematical representation of its parameter exists as product of the internal characteristics and a given material constant. Hence, it suffices that non-locality of the material considers the effects of neighboring points on material characteristics in terms of behavior and in turn possess tremendous impacts to wave modulation and propagation in or on material bodies. However, one aspect of engineering materials that permeates the structural, civil, and construction industries is the composite material. This is because of their adequate and quality usage in engineering applications. For instance the fibre-reinforced composites Spencer [1], pose a great advantage in this regard due to its awesome mechanical quality in terms of its high tensile strength and even its weightlessness during usage. Thus, investigation of wave propagation on these composites are duly needed for useful insights and even more so, if non-locality Barak et al. [2] is involved.

In furtherance to this, Scientists involve in interior and exterior examination of waves in materials, employ wholesome theories- mathematical, experimental and inferential techniques to connect the dots necessary to gain useful information on these materials. However, these solid half-spaces are typically of two categories: isotropic and anisotropic materials. The two forms could be homogeneously and non-homogeneously characterized. These non-homogeneity depends on how environmental factors like stress exerts on the homogeneous material during deformation. When growth and decay of the material parameters occurs during wave propagation or deformation of elastic materials, inhomogeneity of the material creeps into the model problem Barak et al. [3]. More so, researchers in the area of applied and engineering sciences have consistently involve their investigation of wave propagation and modulation using both Isotropic and Anisotropic material despite the fact that the isotropic cases on composites might not give adequate and clear information about the mechanical behavior of the media sought for than the anisotropic materials which are majorly classed into composites and with great mechanical properties that depicts considerable behavioral accuracy during investigation.

Nevertheless, composites might also not give due and rightful prediction of its behavior if accurately not modeled by employing necessary mechanical, internal and external factors. Some of these mechanical characterizations like the mechanical impedance, Singh [6] which act like a resistance to the motion of acoustic energy aid wave analysis. Thus, it typically signifies the quotient of a generalized force to a generalized rate of change of displacement with respect to time; thereby demonstrating significant insights into material stiffness, damping, and inertial make ups. This further assert that mechanical impedance usually, are appropriated in some model problems at the boundary along with non-planar (corrugated boundary) conditions Asano [4] to aid the introduction of complex dynamics of the systems across surfaces and interfaces of materials and thus, giving soothing understanding of wave propagation other than just in planar bodies. Amplitude of corrugation on boundary surfaces of a material amounts to the height of the ridges or undulations on it boundary surface. It can be quantified via experiments such as the atom scattering or analysis of diffraction information. Surface corrugation amplitude is not just useful only to solid mechanics but finds good application in fluid mechanics where it influences boundary layer stability, drags, and even the energy landscape of the boundary surfaces. It is often in comparison with other characteristic dimensions like the thickness of the boundary layer displacement to gain insights about its relative magnitude and effects on bodies. A simple corrugation surface in sinusoidal form is mathematically represented by the use of the trigonometric Fourier functions of sine or cosine where its amplitude suffices as the coefficient of the cosine function, as the case may be. Variable amplitudes of corrugation of the medium boundary could connote a lot of information about the wave motion; ranging from intensity, sources, characteristics of the material it came from, and the contacts with barriers.

Consequently, research is still ongoing and models are eventually developed to add to the existing body of knowledge as far as impedance and corrugated conditions across interfaces and along surfaces of materials exists. Thus, it's on record that several existing contributions has been made as well by authors in the literatures on impedance, corrugated and inhomogeneous wave propagations or modulations with other physical parameter incorporations. Singh et al. [9-11] hinged his studies on qP-wave at a corrugated interface for two different initial stress elastic half-space material, and influence of corrugated surfaces reinforcement, hydrostatic stress, inhomogeneity and anisotropy on Love type wave propagation and also on the effect of loose bonding and wavy conditions on Rayleigh wave propagation. In a similar vein, Das et al. [12] opined their investigation waves on inhomogeneous material gravity effects while Abd-Alla et al. [13] considered on the influence of rotation of the medium on a non-homogeneous infinite elastic cylinder of orthotropic material exhibiting magnetic impacts. In addition, Chattopadhyay et al [14] dealt on the dispersion of Love wave by considering imperfection in the thickness of a heterogeneous material While Roy et al [16] developed a model to study the propagation and reflection of plane waves in a rotating magneto-elastic fibre-reinforced semi space with surface stress. Similarly, Singh et al [17]; Gupta et al. [18]; Nirwal et al. [19]; Anya et al. [20-23] investigated on magnetic effects on surface waves in a rotating non-homogeneous half-space, wave velocity in bedded piezo-structure with flexoelectric effect with different boundary types, grooved-impedance boundary conditions and non-local effects, as the case may be. Likewise, Maleki et al. [24] opined a model which determine the impactt of various geometrical aspects on horizontal impedance. Also, Chowdhury et al. [25] led a study to investigate the dispersion of Stoneley waves through an imperfect interface of two hydrostatic stressed MTI media. Singh et al [26-27] investigated Rayleigh surface wave at an impedance boundary of an incompressible micropolar and orthotropic solid. Sahu et al. [28] contributed to the development of the Mathematical analysis of Rayleigh waves at the imperfect boundary between two different media. Thus, it is to be noted also that Giovannini [29] examined the theory of dipole-exchange spin-wave propagation in periodically corrugated films while Rakshit et al. [30-31] made a proposition to analyze stress for the imperfect surface of visco-porous piezoelectric half-space with a load under a motion while Eringin [36-37] developed models for linear theory of non-local elasticity and dispersion of plane waves and nonlocal continuum field theories while Roy et al. [38] investigated and discussed Rayleigh wave in a rotating nonlocal magnetoelastic half-plane. Also, Said et al. [39] conceptualized and examined the effect of initial stress and rotation on a nonlocal Fiber-reinforced thermo-elastic medium with a fractional derivative heat transfer. Undoubtedly, we witnessed that all the above examined literatures posited or opined their researches towards a singular or part physical parameter considerations which shows innovation and new ideas in the body of knowledge. However, they fall short of performing or giving analysis in joint physical parameters' representation occasioned in the current examination where variable amplitudes of corrugated surface and inhomogeneous impedance conditions in the theory of fibre-reinforced elasticity holds sway in triggering new ideas and innovation to the research community.

Despite the literatures posited above, the current investigation is aimed at developing a mathematical model and its analysis to predict the understanding of wave propagation and in particular dispersion of Rayleigh surface wave in a heterogeneous fibre-reinforced half-space. This is achieved through the considerations of the inhomogeneous half-space and the inhomogeneous impedance under variable amplitudes of corrugation of the boundary surface. We employed the stress-strain factors of the considered half-space to analytically derive the equations governing the wave motion on the inhomogeneous material. The equations of motion were non-dimensionalized and thus, yielding more associated equations of motion of the wave necessary to gain insights about the wave modulation in the complex mechanical structure. Through this, the wave displacements and stress distributions were developed and presented. Utilizing the developed variable amplitudes of corrugated boundary along with the inhomogeneous impedance conditions at the boundary, the dispersion of the Rayleigh wave for the model is achieved. Graphical results of the dispersion of the Rayleigh wave for a particular chosen material were depicted using Mathematica 11 software. These graphical results were made in such a way as to ascertain the influences of the interacting physical quantities of wavenumber, variable amplitudes of corrugated parameters, and the inhomogeneous quantity on the dispersion of the Rayleigh wave. We observe that these parameters have tremendous impacts on the surface wave propagation on the material. We hold that special cases of this study are found in the literatures if certain parameters are negligible, for instance - one of the parameters associated with the variable amplitudes of corrugation of the material.

2. The Mathematical Model and Formulations

The mathematical constitutive equations and fields' variables for the stress-strain factor of a homogeneous fibre-reinforced half-space utilized in this investigation and as introduced by Spencer [1] and employed in Anya et al. [32-33] are thus presented:

$$\begin{aligned}\sigma_{ij} &= \lambda\varepsilon_{kk}\delta_{ij} + 2\mu_T\varepsilon_{ij} + \alpha(r_k r_m \varepsilon_{km}\delta_{ij} + \varepsilon_{kk} r_i r_j) + 2(\mu_L - \mu_T)(r_i r_k \varepsilon_{kj} + r_j r_k \varepsilon_{ki}) \\ &\quad + \beta(r_k r_m \varepsilon_{km} r_i r_j), \quad i = j = k = m = 1,2,3.\end{aligned} \tag{1}$$

The fields' and physical parameters in the above Eq. (2.1), that is, σ_{ij} entails the stress tensor, ε_{ij} represent the strain tensor, u_i gives the displacement vector, λ is the well-known Lames quantity, (α, β, $(\mu_L - \mu_T)$) denotes fibre-reinforced physical constants, and δ_{ij} entails the Kronecker–delta function. Thus, a mathematical relation of the strain is denoted as; $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ while $\vec{r} = (r_1, r_2, r_3)$ gives the fibre-reinforcement such that we choose $\vec{r} = (1,0,0)$ as the fibre-reinforced directions of the material utilized in this investigation. On this note, the governing dynamical equations of motion on the homogeneous fibre-reinforced half-space is presented:

$$\sigma_{ij,j} = \rho\ddot{u}_i \tag{2}$$

We utilized Einstein's summation indices and where index after comma appears, represents partial rate of change with respect to space coordinate and superscript dot gives partial derivative with respect to time. Recall that our mathematical analysis is stated to hinge in the plane. This means that the displacements $u_1 \neq u_2 \neq 0$ at any rate of change in space coordinates and time at this instance. In addition, taking cognizance of our initial intent of Rayleigh wave dispersion on inhomogeneous medium, it is sufficient to allow the deformation of the elastic parameters in the homogeneous fibre-reinforced solid to grow or decay during the action of stress on the material as occasioned by the wave. This is taking in such a way that the rate of change of these parameters during the deformation is to be proportional to its value at that instance, and in order to introduce the inhomogeneity in the problem formulations. Thus, the elastic parameters and modules, and the density of the fibre-reinforced half-space take the form: $\mu_T = \mu_{T_0} e^{-mx_2}, \beta = \beta_0 e^{-mx_2}$,

$\lambda = \lambda_0 e^{-mx_2}, \alpha = \alpha_0 e^{-mx_2}, \mu_L = \mu_{L0} e^{-mx_2}$ and $\rho = \rho_0 e^{-mx_2}$, Khan et al. [5] and Munish et al. [8]. In the given relations denoting decay of the material parameters, m simply defines the inhomogeneity of the fibre-reinforced material.

More so, when these inhomogeneous parameters are taken into account in Eqs. (1-2), we derive the component equations of motion of the wave as follows:

$$(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)u_{1,11} + (\alpha + \lambda + \mu_L)u_{2,21} + \mu_L u_{1,22} - m\mu_T\left(u_{1,2} + u_{2,1}\right) = \rho\ddot{u}_1, \tag{3}$$

$$(\alpha + \lambda + \mu_L)u_{1,12} + \mu_L u_{2,11} + (\lambda + 2\mu_T)u_{2,22} - m\left(\lambda + \alpha\right)u_{1,1} - m\left(\lambda + 2\mu_T\right)u_{2,2} = \rho\ddot{u}_2, \tag{4}$$

$$\mu_L u_{3,11} + \mu_T u_{3,22} - m\mu_T u_{3,2} = \rho\ddot{u}_3. \tag{5}$$

Also, we can suppress the coefficients in Eqs. (3-5) by denoting them with given quantities and thus, resulting to the equations below:

$$C_1 u_{1,11} + C_2 u_{2,21} + C_3 u_{1,22} - mC_4\left(u_{1,2} + u_{2,1}\right) = \rho\ddot{u}_1, \tag{6}$$

$$C_2 u_{1,12} + C_3 u_{2,11} + C_5 u_{2,22} - mC_6 u_{1,1} - mC_7 u_{2,2} = \rho\ddot{u}_2, \tag{7}$$

$$C_3 u_{3,11} + C_4 u_{3,22} - mC_4 u_{3,2} = \rho\ddot{u}_3 \tag{8}$$

$C_1 = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta),\ C_2 = (\alpha + \lambda + \mu_L), C_3 = \mu_L,\ C_4 = \mu_T,\ C_5 = C_7 = (\lambda + 2\mu_T),\ C_6 = \left(\lambda + \alpha\right).$

If we put $m = 0$ into Eqs. (6-8), the homogeneous solid half-space for the dynamic equations of the wave is achieved. For convenience, let us introduce the following dimensionless variables $(x_1', x_2', u_1', u_2') = c_0(x_1, x_2, u_1, u_2),\ c_0^2, = C_1/\rho, (t') = c_0^2$ t, $\sigma_{ij}' = \sigma_{ij}/\rho c_0^2$, and use all into Eqs (6-8) and subsequently removing the sign "$'$" from the equations, we obtain a non-dimensionlized dynamic equations of the wave as:

$$u_{1,11} + C_{12}u_{2,21} + C_{13}u_{1,22} - mC_{24}\left(u_{1,2} + u_{2,1}\right) = \ddot{u}_1, \tag{9}$$

$$C_{12}u_{1,12} + C_{13}u_{2,11} + C_{15}u_{2,22} - mC_{26}u_{1,1} - mC_{27}u_{2,2} = \ddot{u}_2, \tag{10}$$

$$C_{13}u_{3,11} + C_{14}u_{3,22} - mC_{24}u_{3,2} = \ddot{u}_3. \tag{11}$$

$(C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}) = ((C_2, C_3, C_4, C_5, C_6, C_7)/C_1),\ (C_{24}, C_{26}, C_{27}) = (C_{14}, C_{16}, C_{17})\rho^{1/2}/C_1^{3/2}.$

3. Normal mode Analysis and Theoretical Results

In this section, we make use of the normal mode method otherwise called the harmonic solution approach to derive the analytical solutions (closed-form solutions) of the normal stress, shear stress and displacements of the Rayleigh wave on the variable amplitudes of the corrugated boundary surface of an elastic and heterogeneous fibre-reinforced half-space. Utilizing the fact that this normal mode approach be applied, the components of the displacements and the stresses of the wave on the material are thus presented:

$$u_i = (\hat{u}_i(x_2))e^{\omega t + ibx_1}, i = 1,2. \tag{12}$$

Putting Eq (12) into the Eqs (9-11) results to three derived ordinary differential equations (ODEs) which are in the direction of the x_2 coordinate of space as thus given below:

$$(C_{13}D^2 - mC_{24}D - b^2 - \omega^2)\hat{u}_1 + (iC_{12}bD - mC_{24}bi)\hat{u}_2 = 0, \tag{13}$$

$$(iC_{12}bD - mbiC_{26})\hat{u}_1 + (C_{15}D^2 - mC_{27}D - C_{13}b^2 - \omega^2))\hat{u}_2 = 0, \tag{14}$$

$$(C_{14}D^2 - mC_{24}D - C_{13}b^2 - \omega^2))\hat{u}_3 = 0. \tag{15}$$

Remember that D^2 in the above equations stipulates second order ordinary derivative in the direction of x_2 coordinate of space. And thus, considering a non-trivial solution of Eqs. (13-15), that is, for $(\hat{u}_1, \hat{u}_2) \neq 0$, the determinant of Eqs. (13-14) equal zero. This lead to obtaining the quartic characteristics equations below in D such that $\hat{u}_1, \hat{u}_2$ takes on as the dependent variables and x_2 the independent variable of the ODE. A keen look at the equations above show that Eq. (15) is independent of $\hat{u}_1, \hat{u}_2$ and Eqs. (13-14) are coupled equations and without the $\hat{u}_3$ associated displacement component of the wave on the material. This is avoided in forming the quartic equation or the characteristic equation given below. The reason being that our discussion of this investigation is centered in a plane geometry which gives a 2-D analysis.

$$(a_{11}D^4 + a_{12}D^3 + a_{13}D^2 + a_{14}D + a_{15})(\hat{u}_1, \hat{u}_2) = 0. \tag{16}$$

The coefficients of the characteristic equation i.e. in Eq. (16), $a_{1i}, i = 1,2,3,4,5$ (See Appendix) are termed complex coefficients that involves the physical constants of the nonhomogeneous fibre-reinforced material. Now, if we presume that $\eta_i, i = 1,2,3,4$ be real positive roots of Eq. (3.5) and by the applicability of the normal mode approach, we deduce $\hat{u}_1, \hat{u}_2$ in the form:

$$(\hat{u}_1, \hat{u}_2) = (\mathrm{K}_n, \mathrm{K}_{1n})e^{-\eta_n x_2} n = 1,2,3,4. \tag{17}$$

In addition, the coefficients in the above equations i.e. K_n and K_{1n} above are dependent on the wavenumber b in the direction of x_1 and ω as the complex frequency linked with the surface wave modulation on the nonhomogeneous fibre-reinforced solid half-space. Employing Eq (17) into Eqs. (13-14), the function K_{1n} is constructed below and which depends on K_n:

$$\mathrm{K}_{1n} = N_{1n}\mathrm{K}_n, \tag{18}$$

$N_{1n} = h_{1n} / h_{2n}$, $h_{1n} = (-\omega^2 + C_{13}\eta_n^2 + mC_{24}\eta_n - b^2 + (iC_{12}b\eta_n + mbiC_{26}))$, $h_{2n} = C_{15}\eta_n^2 + mC_{27}\eta_n - C_{13}b^2 - \omega^2 + (iC_{12}b\eta_n + mbiC_{24})$.

$n = 1,2,3,4.$

Following this, complete solutions of the components of the horizontal and normal displacements, and normal and shear stresses are presented:

$u_1 = \mathrm{K}_n e^{-\eta_n x_2 + \omega t + ibx_1}$, $u_2 = N_{1n}\mathrm{K}_n e^{-\eta_n x_2 + \omega t + ibx_1}$, $\sigma_{11} = \{ib - \eta_n N_{1n} C_{16}\}\mathrm{K}_n e^{-(\eta_n + m)x_2 + \omega t + ibx_1}$,

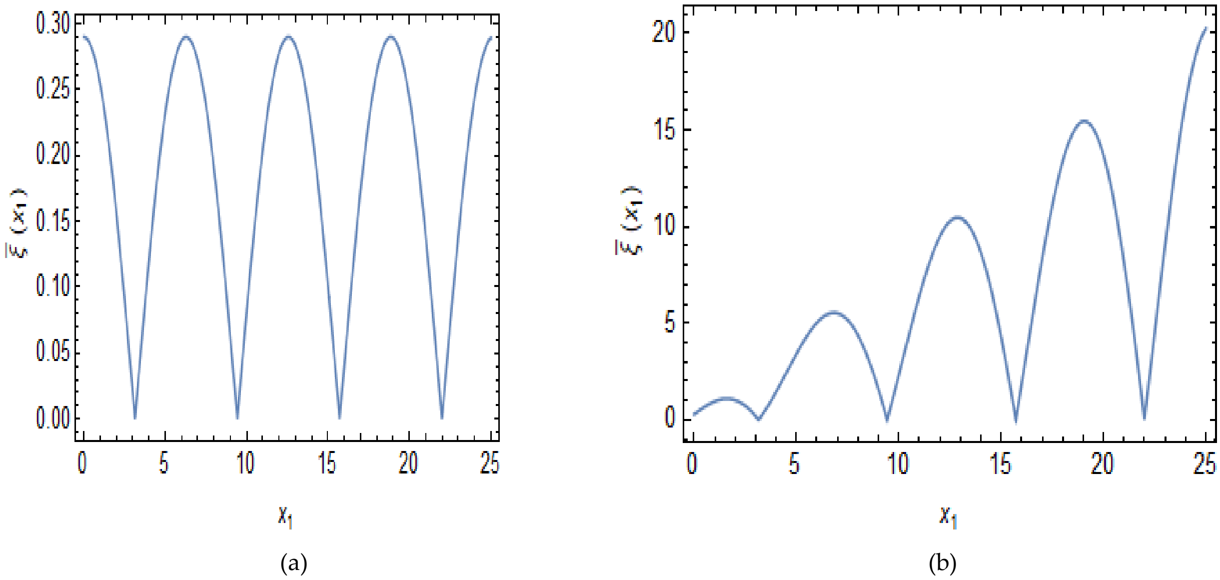
$\sigma_{22} = \{ibC_{16} - \eta_n N_{1n} C_{17}\}\mathrm{K}_n e^{-(\eta_n + m)x_2 + \omega t + ibx_1}$, $\sigma_{12} = (ibN_{1n} - \eta_n)C_{13}\mathrm{K}_n e^{-(\eta_n + m)x_2 + \omega t + ibx_1}$ $\sigma_{21} = C_{13}(ibN_{1n} - \eta_n)\mathrm{K}_n e^{-(\eta_n + m)x_2 + \omega t + ibx_1}$.

$n = 1,2,3,4.$.

4. Impedance and Variable Amplitudes of Corrugation of the Half-space and Dispersion Relation

Formulations involving the variable amplitude of corrugation alongside the impedance (inhomogeneous impedance) condition defines this section. On this note, Asano [4] presented in their work a corrugated condition on a material by utilizing the trigonometric Fourier series $x_2 = \xi(x_1)$ as a periodic function which do not depend on the x_3-axis. They represented $x_2 = \xi(x_1)$ to be in the form: $\xi(x_1) = \xi_l e^{ilbx_1} + \xi_{-l} e^{-ilbx_1}, l = 1,2,3,4,...$ Here, ξ_l and ξ_{-l} entails the Fourier expansion coefficients and l is the series expansion order. Asano found the expansion of this series in Fourier sine and Fourier cosine terms and thereafter use the term $\xi(x_1) = a\cos bx_1$ as the representation of the corrugated surface boundary in his model. In their formulations, a means the constant amplitude of corrugation and b the wave number associated with wave on the corrugated surface having wavelength as $2\pi / b$. Nevertheless, we are concerned with the scenario where the amplitude of corrugation of the material is dependent on the horizontal coordinate of space instead of a constant such that the wavelength of wave on the corrugated surface becomes π / b. By this, half of the wavelength obtained by Asano in his formulations for a non-variable amplitude of the wave is achieved. Following this, we need to reformulate the constant amplitude to variable amplitude, such that this would reflect in our current model. This is carried out by redefining the amplitude of the corrugated surface such that $\bar{\xi}_1^{\pm} = (a + cx_1)/2$, and $\bar{\xi}(x_1) = \bar{\xi}_l e^{ilbx_1} + \bar{\xi}_{-l} e^{-ilbx_1}, l = 1,2,3,4,...$, via which the Fourier cosine term and Fourier sine expansions be presented as: $\bar{\xi}(x_1) = (a + cx_1)\cos bx_1 + F_2 \cos 2bx_1 + I_2 \sin 2bx_1 + ... + F_l \cos lbx_1 + I_l \sin lbx_1$.

Also, $\bar{\xi}_l^{\pm} = (F_l + I_l)/2$, $l = 2,3...$ Consequently, we then presume that the corrugation takes the form: $\bar{\xi}(x_1) = (a + cx_1)\cos bx_1$. Such that $(a + cx_1)$ gives the variable amplitudes of the corrugation surface and b the wavenumber associated with the variable corrugated boundary. However, a, c are terms associated with the variable amplitudes. If $c = 0$, we retrieve constant amplitude of the corrugated surface associated with associated with Asano [4] model. In other to depict these two occurrences of corrugations, that is, the corrugation with constant amplitudes; $\xi(x_1) = a\cos bx_1$ and that with variable amplitudes; $\bar{\xi}(x_1) = (a + cx_1)\cos bx_1$, Figure 1 present the following graphical illustrations below:



(a) (b)

Figure 1. (a) Uniform amplitude of corrugation $\xi(x_1)$ versus (b) Variable amplitudes of corrugation $\bar{\xi}(x_1)$, with respect to x_1 in meters.

This entails that the nature of surface corrugation would enhance the dispersion of Rayleigh wave and its velocity especially where the variability of its amplitudes along the boundary interactions plays an important role.

Now, we present as follows the inhomogeneous boundary condition on the impedance considering the fact that the material is heterogeneous with variable amplitudes of corrugation:

(i) $u_1 = 0, \quad u_2 = 0,$ at $x_2 = \bar{\xi}(x_1)$, for all coordinate x_1 and time *t.*

(ii) The normal stress w.r.t $x_2 = \bar{\xi}(x_1)$ yields the following condition: $\sigma_{22} - \bar{\xi}'(x_1)\sigma_{21} + \omega \bar{Z}_2 u_2 = 0,$

(iii) The shear stress holds true if $\sigma_{12} - \bar{\xi}'(x_1)\sigma_{11} + \omega \bar{Z}_1 u_1 = 0,$ for all x_1 and time *t,* Anya et al. [32-33] and Azhar et al. [34].

The quantities $\bar{Z}_1$ and $\bar{Z}_2$ are the nonhomogeneous impedance parameters, Anya et al. [32-33] and Ailawalia et al. [7]. Taking into account our formulations on heterogeneous medium where the deformation of the material infuses non-homogeneity into the system, we thus redefine our impedances in the form $(\bar{Z}_1, \bar{Z}_2) = (Z_1, Z_2)e^{-mx_2}$. Recall that the quantities Z_1, Z_2 are homogeneous but the deformation of the material has made them to change cause into nonhomogeneous parameters. This is such that when $m = 0$ in $\bar{Z}_1$ and $\bar{Z}_2$ recovery of the quantities $Z_1,$ and Z_2 are undoubtedly achieved at the boundary and thus, giving rise to homogeneous impedance, say. Thus, these presumptions imply the following set of four equations in (a) below:

(a) Inhomogeneous impedance boundary condition considering fibre-reinforced inhomogeneous material:

$$\mathrm{K}_n = 0, \tag{19}$$

$$N_{1n}\mathrm{K}_n = 0, \tag{20}$$

$$\begin{aligned} &\{ibC_{16} - \eta_n N_{1n} C_{17}\}e^{-(\eta_n)\xi(x_1)}\mathrm{K}_n + [(a + cx_1)b\sin bx_1 - c\cos bx_1]\{(ibN_{1n} - \eta_n)C_{13}\}e^{-(\eta_n)\xi(x_1)}\mathrm{K}_n \\ &\qquad + \{\omega N_{1n} Z_2 \mathrm{K}_n\}e^{-(\eta_n)\xi(x_1)}\mathrm{K}_n = 0, \end{aligned} \tag{21}$$

$$\begin{aligned} &\{\{ibN_{1n} - \eta_n\}C_{13}\mathrm{K}_n + [(a + cx_1)b\sin bx_1 - c\cos bx_1]\{ib - \eta_n N_{1n} C_{16}\}\mathrm{K}_n \\ &\qquad + \{\omega Z_1\}\}\mathrm{K}_n e^{-(\eta_n)\xi(x_1)} = 0, \end{aligned} \tag{22}$$

Here, $n = 1,2,3,4.$ A non-trivial solution of Eqs. (19-22) gives the dispersion relation of the Rayleigh wave on the inhomogeneous fibre-reinforced medium. That is for $\mathrm{K}_n \neq 0,$ the determinant $|\mathrm{K}_{ij}| = 0, i = j = 1,2,3,4$ entails the derived analytical solution of the dispersion $|\nabla|$ of the Rayleigh wave for an inhomogeneous conditions on the impedance. Note that a special case of this dispersion $|\nabla|$ could ensue if we take $c = 0$ in Eqs. (19-22) thereby yielding results for constant amplitudes of corrugation on the inhomogeneous fibre-reinforced material.

5. Results and Discussion

In this section, the effects of the physical quantities of wavenumber b, the parameters (a, c) associated with the variable amplitudes of corrugation, impedance $Z_i, i = 1,2$ and the heterogeneity m on the distribution of the dispersion of the Rayleigh wave for the inhomogeneous fibre-reinforced half-space are ascertained. These are achieved and presented in Figures 2-7 through the use of the fibre-reinforced physical constants given by Othman et al. [35] and some other parameters below:

$\lambda = 3.76\times10^9\,kg\,m^{-1}s^{-2};\mu_L = 7.86\times10^9\,kg\,m^{-1}s^{-2};\mu_T = 2.86\times10^9\,kg\,m^{-1}s^{-2};\rho = 7800kg\,m^{-3};\ \alpha = \text{-}1.78\times10^9\,kg\,m^{-1}s^{-2};$
$\beta = 2\times10^9\,kg\,m^{-1}s^{-2};\ \omega = (0.8\ -\ 0.5i)rad/s; t = 0.2s; b = 0.6; m = 6; Z_1 = 0.05; Z_2 = 0.07; c = 0.0007; a = 0.290.$

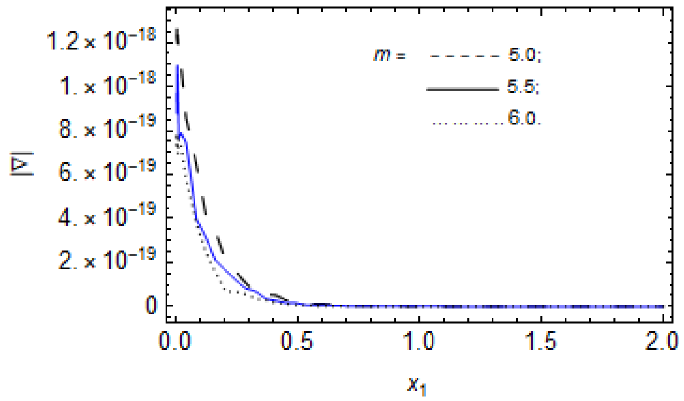


Figure 2. Impact of inhomogeneous parameter m on the distribution of the dispersion $|\nabla|$ of Rayleigh wave against x_1 in meters, considering inhomogeneous impedance

Figure 2 gives the effects of the heterogeneous parameter m on the dispersion $|\nabla|$ of the Rayleigh wave with respect to x_1 coordinate, considering inhomogeneous impedance and fixed physical parameters of wavenumber b, (a and c) associated with variable amplitudes of corrugation and the impedance $Z_i, i = 1,2$ on the inhomogeneous fibre-reinforced half-space. On this note, we witness a downward trend when the inhomogeneous parameter m increase within the domain $0 < x_1 < 0.7$ before uniform behaviors of the dispersion $|\nabla|$ ensues. Thus, we can physically adduce that this is feasible considering non-uniformity of the fibre material during deformations and hence yielding variations and uniformity in modulations along certain domains of the wave path. The maximum amplitude of the dispersion $|\nabla|$ of the Rayleigh wave occur near the extended length of the material especially when the heterogeneous parameter $m = 5$ while its minimum value can be found when $m = 6$ and near $x_1 = 0$. This means that as the heterogeneous parameter becomes small on the material, the material characteristics or make-ups tends to allow more surface wave propagation on it.

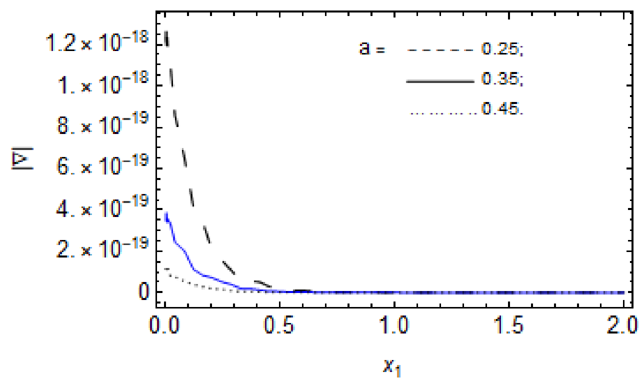


Figure 3. Impact of a associated with the variable amplitude of corrugation on the distribution of the dispersions $|\nabla|$ of Rayleigh wave against x_1 in meters, considering inhomogeneous impedance

Consequently, Figure 3 demonstrate the effect of the parameter a associated with variable amplitudes of corrugation on the distribution of the dispersions $|\nabla|$ of Rayleigh wave with respect to x_1 coordinate. This holds only by considering the inhomogeneous impedance on the inhomogeneous fibre-reinforced half-space and when the physical parameters of wavenumber b, c associated with variable amplitudes of corrugation, impedance $Z_i, i=1,2$ and heterogeneity m of the material are applied in a steady manner on the material. It is observed that the parameter a associated with variable amplitudes of corrugation in Figure 4 shows a clear downward trend on the dispersion of the Rayleigh wave in the domain $0<x_1<0.7$ when its value increases before uniform behaviors sets in along the extended length of the material. Also, the maximum amplitude of dispersion $|\nabla|$ of the wave occur when the parameter a associated with the variable amplitude of corrugation is small say, $a=0.25$.

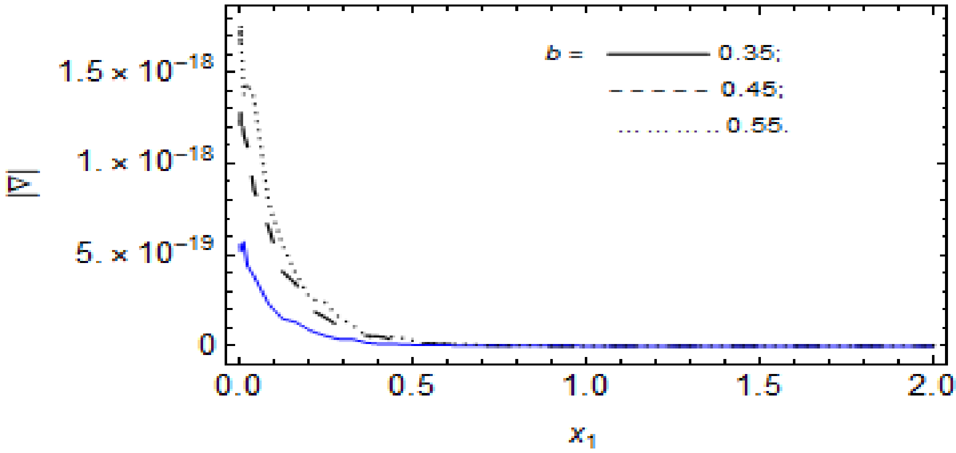


Figure 4. Impact of b associated with the variable amplitude of corrugation on the distribution of the dispersions $|\nabla|$ of Rayleigh wave against x_1 in meters, considering inhomogeneous impedance

Subsequently, Figure 4 showcase the effect of the wavenumber b on the distribution of the dispersions $|\nabla|$ of Rayleigh wave as against x_1 coordinate vis-à-vis the consideration of inhomogeneous impedance on the inhomogeneous medium when the constant applications of the quantities of impedance $Z_i, i=1,2,$ inhomogeneous parameter m, variable amplitude of corrugated parameters (a, c) on the inhomogeneous solid half-space are maintained. On this note, we observed that the wavenumber b demonstrates an increasing behavior on the dispersion of the wave in the domain $0<x_1<0.7$ when its value is increased and after which a uniform modulation sets in. In fact, high wavenumber will produce a considerable large amplitude of dispersion and at this point yielding its maximum value of dispersion. Decreasing the value of the wavenumber have shown that Figure 4 has considerable oscillatory tendency of the dispersion of the Rayleigh wave and this could be visualized in the domain $0<x_1\leq 0.5$. Thus, this physically connotes that the wavenumber due to the corrugation would impact the velocity and attenuation of the surface wave propagating on the material

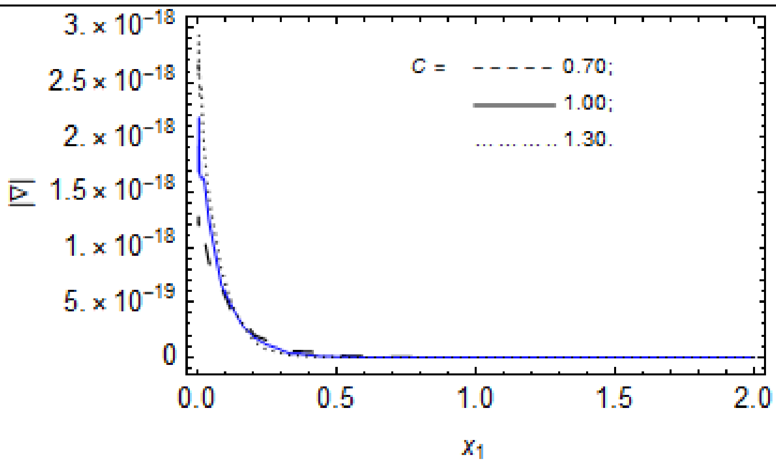


Figure 5. Impact of c associated with the variable amplitude of corrugation on the distribution of the dispersion $|\nabla|$ of Rayleigh wave against x_1 in meters, considering inhomogeneous impedance

Similarly, in Figure 5 above, the influence of c linked with the variable amplitudes of corrugation on the distribution of the dispersion $|\nabla|$ of Rayleigh wave as against x_1 coordinate is demonstrated. This is such that the inhomogeneous impedance on the inhomogeneous half-space and the physical constants of impedance $Z_i, i = 1,2,$ inhomogeneous parameter m, variable amplitude of corrugated parameter a on the inhomogeneous solid half-space are unchanged. Figure 6 demonstrate a very clear mix behavior (in terms of increase and decrease) of the dispersion of the Rayleigh wave on the material for an increase in the parameter c associated with variable amplitudes of corrugation of the material in the domain $0.2 < x_1 < 0.6$ before uniform before ensues. Also, with the domain $0 < x_1 < 0.2$ the dispersion relation increases for increase in c linked with the variable amplitudes of corrugation. We observe that the maximum dispersion occurs near the origin $x_1 = 0$ and $c = 1.3$ (that is large). While the minimum occurs near the vanishing region of the length of the material when is small say, $c = 0.7$. That is, c produces the minimum value of dispersion when its value is small on the material whilst noting uniform exhibition on the dispersion $|\nabla|$ for an upward variation in its value and along extended length of the material.

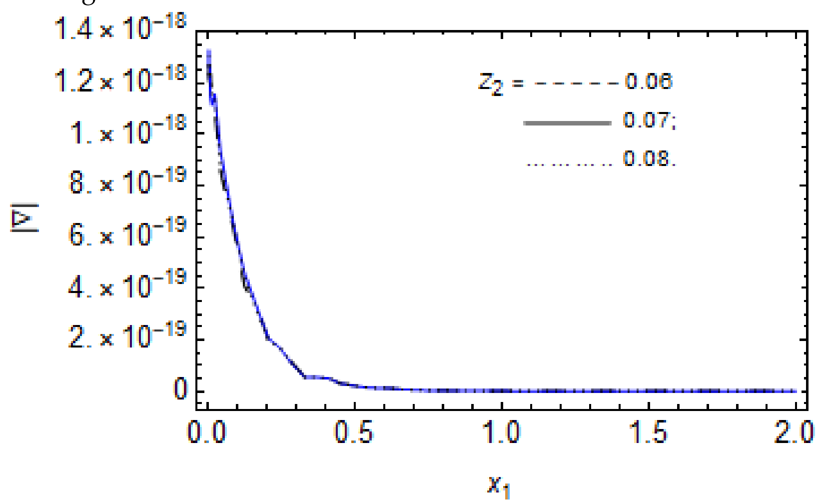


Figure 6. Impact of impedance Z_2 on the distribution of the dispersion $|\nabla|$ of Rayleigh wave against x_1 in meters, considering inhomogeneous impedance

Figure 6 depicts the impact of impedance Z_2 on the distribution of the dispersions $|\nabla|$ of Rayleigh wave with respect to x_1 coordinate, considering inhomogeneous impedance on the inhomogeneous medium and following that the wavenumber b, horizontal impedance Z_1, inhomogeneous parameter m, parameters (a,c) associated with the variable amplitudes of corrugation applied on the solid half-space are unchanged. This figure shows that increase in the normal impedance Z_2 leads to a near negligible or uniform behavior of the dispersion $|\nabla|$ as the wave propagates. This exhibition is linked to the resistant nature of mechanical impedance on material bodies where movement of acoustic energy on the material is slowed. The maximum value of dispersion lie near the origin while the minimum lies near the vanishing point of the wave propagation or say $x_1 = 2$ at this instance. This have shown that the impedance parameter demonstrated a resistance to the amplitude of the wave and thus impacting its velocity and attenuation or behavior which in turn impact the dispersion effects of the Rayleigh wave.

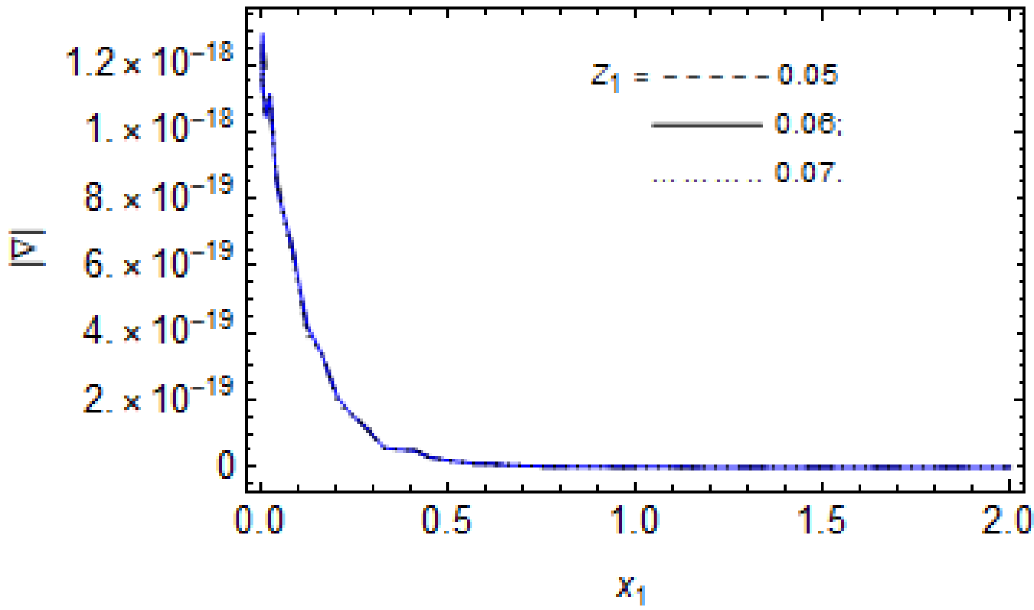


Figure 7. Impact of impedance Z_1 on the distribution of the dispersion $|\nabla|$ of Rayleigh wave against x_1 in meters, considering inhomogeneous impedance

In addition, Figure 7 showcases the characteristics of impedance Z_1 on the distribution of the dispersions $|\nabla|$ of Rayleigh wave as against the x_1 coordinate considering inhomogeneous impedance on the inhomogeneous medium and such that the dimensionless nonlocal quantity, wavenumber b, normal impedance Z_2, inhomogeneous parameter m, parameters (a,c) associated with the variable amplitudes of corrugation applied to the solid half-space remain in steady state. Hence, Figure 7 entails that increase in the horizontal impedance Z_1 yields to a clear negligible or uniform behavior of the dispersion $|\nabla|$ of the wave on the inhomogeneous solid. We note that the minimum value of the dispersion $|\nabla|$ lie near $x_1 = 2$ while the maximum is reached near the origin of the length of the material. Thus, we adduce that similar analysis is obtainable is in Figure 6 above.

6. Conclusions

The fibre-reinforced theory of elasticity coupled with the prevailing physical quantities of variable amplitude of corrugation and inhomogeneous impedance boundary conditions played host in the current investigation of dispersion of Rayleigh wave. The equations of motion were derived by employing the constitutive stress-strain relations of a fibre-reinforced material whilst incorporating effects of heterogeneity. Analytical solution of the displacements and stresses occasioned by the surface waves on the material were deduced using the normal mode solution method after non-dimensionalization of the equation of motion. On this note, the dispersion of the surface wave (Rayleigh wave) was analytically derived by utilizing suitable variable amplitudes of corrugation and inhomogeneous impedance boundary conditions. In trying to decipher the information contained therein in the system about the dispersion of the Rayleigh wave, computational solution as a depiction of the various effects of the considered physical parameters of wavenumber, corrugation quantities (parameters of variable amplitudes of corrugation), and impedance were graphically shown. Following this, we observe that:

1. A downward trend on the dispersion of the wave ensues when the inhomogeneous parameter m increase within certain domain of the horizontal coordinate before uniform behaviors of the dispersion of the Rayleigh wave distribution were encountered.
2. The parameter a associated with variable amplitudes of corrugation cause a downward trend to the dispersion of the Rayleigh wave when its value increases on the solid. While its counterpart c demonstrate a very clear increase in behavior on the dispersions of the Rayleigh wave on the material when increased within certain domains of the horizontal coordinate or length of the material.
3. The wavenumber produce increase in behavior of the dispersion of the surface wave when increased. This means that the higher the wavenumber, the higher the dispersion of the surface wave distribution on the material. However, we noted an oscillatory behavior along the length of the material.
4. Both the normal and horizontal impedance conditions produce uniform or negligible behavior of the dispersion of the wave when increased on the material. This signals resistant to motion or propagation of the Rayleigh wave thereby impacting the velocity and attenuation coefficients.

Be that as it may, special cases of this study and its discussion ensues if certain parameters are negligible. That is, if we take parameters of the variable amplitudes equal to zero, we have cases found in the literature, that is, when we neglect one of the variable amplitudes of corrugations, say $c = 0$, results for Asano [4] formulations of corrugation with constant amplitude of corrugation on the material are gotten. Thus, we adduce that this present research work would be of great interest to researchers examining problems of waves on surfaces of materials especially on that which involves heterogeneous fibre-reinforcement, inhomogeneous impedance and variable corrugated surfaces, amongst others. As stated earlier, variable amplitudes of corrugation of a material would give more information about the wave motion which ranges from the intensity, sources, characteristics of the material it came from, and the contacts with barriers. Hence, it's evident that combined physical interactions of the considered parameters (variable amplitudes of corrugation and heterogeneous impedance parameters) would aid good analysis to engineering problems associated with surface waves with regards to non-destructive testing, production of actuator devices and sensors.

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Nomenclatures

b = Wavenumber

a, c = Parameters associated with variable amplitude of corrugation

σ_{ij} = Stress tensor.

ε_{ij} = Strain tensor.

u_i = Displacement vector.

δ_{ij} = Kronecker-Delta function.

λ = Lame's constant.

$(\alpha,\ \beta,\ (\mu_L - \mu_T))$ = Fibre-reinforced parameters.

ρ = Density

x_i = coordinates.

Z_1, Z_2 = impedance parameters

Appendix

$a_{11} = (C_{13}C_{15});$

$a_{12} = -m(C_{15}C_{24} + C_{13}C_{27});$

$a_{13} = (-i^2 b^2 C_{12}^2 - b^2 C_{15} - \omega^2 C_{15} + m^2 C_{24}C_{27} - C_{13}(\omega^2 + b^2 C_{14}));$

$a_{14} = m(i^2 b^2 C_{12} C_{26} + C_{27}(b^2 + \omega^2) + C_{24}(\omega^2 + b^2 i^2 C_{12} + b^2 C_{14}));$

$a_{15} = b^2 C_{14}(b^2 + \omega^2) + \omega^2 (b^2 + \omega^2) - b^2 i^2 m^2 C_{24} C_{26}.$

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