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Article

Power transformation design-based calibration scheme of robust measures under stratified sampling

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Abstract: In this article, two modified design-based calibration scheme-type estimators are formulated. The suggested estimators were proposed under robust measure of stratified sampling as information of auxiliary variables is sensitive to outliers on data. The technique such as coefficient of variation, Gini mean diference, Downton method, Probability waited moment, Median of the class, Hodge-Lehmann, first quarter, first decile, second decile, third decile were handle presence of outliers. The metric means square error (MSE) of the suggested calibrated scheme-type estimators are deduced up to first order of approximation by new Tailor series approach. The conditions at which the proposed calibrated scheme-type estimators outperformed existing estimators counterparts are studied numerically using simulated data under two populations, which were generated using Log-normal and Weibull distributions respectively with R scripts. The proposed calibrated scheme-type estimators efficiency in a rows is kept continue up to last population (Weibull) without any exception.

Keywords: calibration, consistency, stratified, outliers or extreme values, robust measures, Langrage multipliers.

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1. Introduction

Sampling method is seriously engaging in calibration since recent decay. The commonly used technique to produce new estimation weights includes the following approach (I) computation of new weights that incorporate with specified auxiliary information and are restricted to calibration problem constrains (II) These weights can be to compute linearly weighted estimate of mean, median, totals, variance and other finite population parameters fulfilling the requirement objective of obtaining nearly unbiased estimate. The technique is used to derived cosmetic estimators (estimators interpretable both as design-based and as prediction-based estimators) (see [1]-[28]).

The calibration technique has also been utilized to develop design-based estimator under different sampling schemes like stratified random sampling, stratified random double sampling, two-stage sampling, etc. In this direction many authors like ([29]-[42]) have proposed estimators and studied their properties for estimating population mean under different calibration constraints in stratified random sampling. [43] obtained calibration weights for population mean by using first and second order moments of auxiliary variable in stratified random sampling. [44] considered estimation of population mean using calibration approach in stratified and stratified double sampling schemes. [45] utilized calibration approach in defining estimators for population variance in stratified random sampling.

Other authors like ([46]-[51]) considered estimation of population mean under two stage sampling schemes using the calibration approach. The focused, in this paper is to suggested calibrated scheme-type in stratified random sampling by utilizing auxiliary variables on some robust statistical measures such as Gini mean difference, Downton method, Probability Waited moment, Median of Class, Hodge Lehman, and Quarter Range, all of which are effect against the presence of outliers in the population and are less susceptible to fluctuations in sampling whenever extreme observations are present as alternatives to [33] and [37] calibration estimators.

2. Basic Definition and Notations

Consider a population $\mathfrak{H}_N = \left(\mathfrak{H}_{N_h}, h = 1,2,3,\ldots,l\right)$ from a stratified non-overlapping heterogeneous with l strata of size $N=\sum_{h=1}^{l} N_h$. Let $y_{hi},\left(i=1,2,3,\ldots,N_{hi}\right)$ be the values of the study variable y, from i^{th} population unit and $x_{hi},\left(i=1,2,3,\ldots,N_{hi}\right)$ be the value the i^{th} unit of the associated auxiliary variable. $\bar{Y}_h = N_h^{-1}\sum_{h=1}^{N_h} Y_{hi}$ $S_{Yh}^2 = \left(n_h-1\right)^{-1}\sum_{i=1}^{N_h}\left(Y_{hi}-\bar{Y}_h\right)^2$, $\bar{X}_h = N_h^{-1}\sum_{h=1}^{N_h} Y_{hi}$ and $C_{xh} = S_{xh}^2/\bar{X}_h$ $\Delta_h = N_h/N$ $S_{Xh}^2 = \left(n_h-1\right)^{-1}\sum_{i=1}^{N_h}\left(X_{hi}-\bar{X}_h\right)^2$ are mean, variance and design weight of population in each stratum for the study and auxiliary variable respectively. $\bar{y}_h = n_h^{-1}\sum_{h=1}^{n_h} y_{hi}$, $s_{yh}^2 = \left(n_h-1\right)^{-1}\sum_{i=1}^{N_h}\left(y_{hi}-\bar{y}_h\right)^2$, $\bar{x}_h = n_h^{-1}\sum_{h=1}^{n_h} x_{hi}$ $R = \bar{X}_h/\bar{Y}_h$, $s_{xh}^2 = \left(n_h-1\right)^{-1}\sum_{i=1}^{N_h}\left(x_{hi}-\bar{x}_h\right)^2$ and $s_{xyh}^2 = \left(n_h-1\right)^{-1}\sum_{i=1}^{N_h}\left(x_{hi}-\bar{x}_h\right)\left(y_{hi}-\bar{y}_h\right)$ are mean, variance of sample in each stratum for the study and auxiliary variables respectively. A random sample of size $n=\sum_{h=1}^{l} n_h$ is selected from the population using SRSWOR. The usual unbiased estimator of the population mean and its variance is presented in Eq (1) and Eq (2) respectively.

$$\bar{y}_{st} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \tag{1}$$

$$Var\left(\bar{y}_{st}\right) = \sum_{h=1}^{l} \Delta_h^2 \left(n_h^{-1} - N_h^{-1}\right) S_{yh}^2 \tag{2}$$

Consider calibration estimator of population mean with chi-square and two linear constraints Eq (3) and Eq (4) in stratified random sampling of the form [33]

$$\bar{y}_S = \sum_{h=1}^{l} \Phi_h^S \bar{y}_h \tag{3}$$

$$\left.\begin{array}{l} U_1 = \sum_{h=1}^{l} \left(\Phi_h^S - \Delta_h\right)^2 / \Delta_h d_h^* \\ s.t \sum_{h=1}^{l} \Phi_h^S \bar{x}_h = \sum_{h=1}^{l} \Delta_h \bar{X}_h \\ \sum_{h=1}^{l} \Phi_h^S = \sum_{h=1}^{l} \Delta_h \end{array}\right\} \tag{4}$$

where Φ_h^S, Δ_h and d_h denotes new weights, calibration weights and suitable chosen scale factor which determined the form of the estimator respectively

$$D_1 = \sum_{h=1}^{l} \frac{\left(\Phi_h^S - \Delta_h\right)^2}{\Delta_h d_h^*} - 2\alpha_a \left(\sum_{h=1}^{l} \Phi_h^S \bar{x}_h - \sum_{h=1}^{l} \Delta_h \bar{X}_h\right) - 2\alpha_b \left(\sum_{h=1}^{l} \Phi_h^S - \sum_{h=1}^{l} \Delta_h\right) \tag{5}$$

Minimization of chi-square distance function between new weight and calibration weights lead to Lagrange function D_1 subject to the linear constraint Eq (4) which enable to obtained new weights and resultant estimator given below

$$\Delta_h^S = \Delta_h + \left(\bar{X} + \sum_{h=1}^{l} \Delta_h \bar{x}_h\right) \left(d_h^* \Delta_h \bar{x}_h \sum_{h=1}^{l} \Delta_h \bar{x} d_h^* - d_h^* \Delta_h \bar{x}_h \sum_{h=1}^{l} \Delta_h \bar{x}_h d_h^*\right) \Big/ \left(\sum_{h=1}^{l} \Delta_h d_h^* \sum_{h=1}^{l} \Delta_h \bar{x}_h^{-2} d_h^* - \left(\sum_{h=1}^{l} \Delta_h \bar{x}_h d_h^*\right)^2\right) \tag{6}$$

Calibration design-based estimator for estimating population mean in combined ratio with associated calibration equation as [38].

$$\bar{y}_{ce} = \sum_{h=1}^{l} \Phi_h^{ce} R \bar{X} \tag{7}$$

$$\text{Min} \left.\begin{array}{l} U_2 = \sum_{h=1}^{l} \left(\Phi_h^{ce} - \Delta_h\right)^2 / \Delta_h d_h^* \\ st \sum_{h=1}^{l} \Phi_h^{ce} \bar{x}_h = \bar{X} \end{array}\right\} \tag{8}$$

as usual Δ_h and d_h are denote selection weights and suitable chosen constant such that the estimator depends upon its choice while Φ_h^{CE} is combined design weight.

$$D_2 = \sum_{h=1}^{l} \frac{\left(\Phi_h^{CE} - \Delta_h\right)^2}{\Delta_h d_h^*} - 2\beta \left(\sum_{h=1}^{l} \Phi_h^{CE} \bar{x}_h - \sum_{h=1}^{l} \Delta_h \bar{X}_h\right) \tag{9}$$

To obtain the new weight and resultant scheme estimator we employed Langrage multipliers to minimized chi-square function subject to constraint

$$\text{Calibrate weight } \Delta_h^{ce} \text{ is } \Phi_h^{ce} = \Delta_h + \frac{\Delta_h d_h^* \bar{x}_h}{\sum_{h=1}^{l} \Delta_h d_h^* \bar{x}_h} \left(X - \sum_{h=1}^{l} \Delta_h \bar{x}_h\right) \tag{10}$$

$$\text{Estimator of } \bar{y}_{ce} \text{ is } \bar{y}_{ce} = \bar{X} \sum_{h=1}^{l} \Delta_h \bar{y}_h \Big/ \sum_{h=1}^{l} \Delta_h \bar{x}_h \tag{11}$$

$$\text{Bias of estimator of } \bar{y}_{ce} \text{ is } B(\bar{y}_{ce}) = \bar{X}^{-1} \sum_{h=1}^{l} \Delta_h^2 \left(n_h^{-1} - N_h^{-1}\right)\left(R S_{xh}^2 - S_{xyh}^2\right) \tag{12}$$

$$\text{MSE of the } \bar{y}_{ce} \text{ is } M\left(\bar{y}_{ce}\right) = \sum_{h=1}^{l} \Delta_h^2 \left(n_h^{-1} - N_h^{-1}\right)\left(S_{yh}^2 - 2R S_{xyh}^2 + R^2 S_{xyh}^2\right) \tag{13}$$

The first to introduced measure of dispersion (coefficient of variation) of the auxiliary variable in calibration, the ch-square distance function between new weight and design weight using formal ideas earlier place to improve the precision of the sample population mean in stratified sampling. The two suggest design calibrate are given below [41]

$$\bar{y}_{RTK1} = \sum_{h=1}^{l} \Phi_h^{RTK1} \bar{y}_h \tag{14}$$

$$\min \left.\begin{array}{l} U_3 = \sum_{h=1}^{l} \left(\Phi_h^{RTK1} - \Delta_h\right)^2 / \Delta_h d_h^* \\ st \sum_{h=1}^{l} \Phi_h^{RTK1} \left(\bar{x}_h + c_{xh}\right) = \sum_{h=1}^{l} \Delta_h \left(\bar{X}_h + C_{xh}\right) \end{array}\right\} \tag{15}$$

$$D_3 = \sum_{h=1}^{l} \frac{\left(\Phi_h^{RTK1} - \Delta_h\right)}{\Delta_h d_h^*} - 2\lambda\left(\sum_{h=1}^{l} \Phi_h^{RTK1}\left(\bar{x}_h + c_{xh}\right) - \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + C_{xh}\right)\right) \tag{16}$$

$$\Phi_h^{RTK1} = \Delta_h + \Delta_h\left(\sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + C_{xh}\right) - \sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + c_{xh}\right)\right)\left(\sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + c_{xh}\right)\right)^{-1} \tag{17}$$

$$\bar{y}_{RTK1} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + C_{xh}\right) \Big/ \sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + c_{xh}\right) \tag{18}$$

Another estimator is defined subjected to constraint with chi-square distance function with also the same design weight

$$\bar{y}_{RTK2} = \sum_{h=1}^{l} \Phi_h^{RTK2} \bar{y}_h \tag{19}$$

$$\left.\begin{array}{c} U_4 = \left(\Phi_h^{RTK2} - \Delta_h\right)^2 / \Delta_h d_h^* \\ st \sum_{h=1}^{l} \Phi_h^{RTK2}\left(1 + \bar{x}_h + c_{xh}\right) = \sum_{h=1}^{l} \Delta_h\left(1 + \bar{X}_h + C_{xh}\right) \end{array}\right\} \tag{20}$$

$$D_4 = \sum_{h=1}^{l} \frac{\left(\Phi_h^{RTK2} - \Delta_h\right)^2}{\Delta_h d_h^*} - 2\gamma\left(\sum_{h=1}^{l} \Phi_h^*\left(1 + \bar{x}_h + c_{xh}\right) - \sum_{h=1}^{l} \Delta_h\left(1 + \bar{X}_h + C_{xh}\right)\right) \tag{21}$$

which give another design weight and estimator as

$$\Delta_h^* = \Delta_h + \Delta_h\left(\sum_{h=1}^{l} \Delta_h\left(1 + \bar{x}_h + c_{xh}\right) - \sum_{h=1}^{l} \Delta_h\left(1 + \bar{X}_h + C_{xh}\right)\right) \Big/ \sum_{h=1}^{l} \Delta_h\left(1 + \bar{x}_h + c_{xh}\right) \tag{22}$$

$$y_{RTK2} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \sum_{h=1}^{l} \Delta_h\left(\sum_{h=1}^{l} \Delta_h\left(1 + \bar{x}_h + c_{xh}\right) - \sum_{h=1}^{l} \Delta_h\left(1 + \bar{X}_h + C_{xh}\right)\right) \tag{23}$$

However, estimator y_{RTK2} and y_{RTK2} are function of coefficient of variation which affected outliers or extreme value

$$B\left(y_{RTK1}\right) = \bar{X}^{-1} \sum_{h=1}^{l} \Delta_h\left(n_h^{-1} - N_h^{-1}\right)\left(R\omega_{ai} S_{xh}^2 - \omega_{ai} S_{xyh}^2\right) \tag{24}$$

$$M\left(y_{RTK1}\right) = \bar{X}^{-1} \sum_{h=1}^{l} \Delta_h\left(n_h^{-1} - N_h^{-1}\right)\left(S_{yh}^2 + R^2 \omega_{ai}^2 S_{xh}^2 - 2R\omega_{ai} S_{xyh}\right) \tag{25}$$

$$B\left(y_{RTK1}\right) = \bar{X}^{-1} \sum_{h=1}^{l} \Delta_h\left(n_h^{-1} - N_h^{-1}\right)\left(R\omega_{bi} S_{xh}^2 - \omega_{bi} S_{xyh}^2\right) \tag{26}$$

$$M\left(y_{RTK1}\right) = \bar{X}^{-1} \sum_{h=1}^{l} \Delta_h\left(n_h^{-1} - N_h^{-1}\right)\left(S_{yh}^2 + R^2 \omega_{bi}^2 S_{xh}^2 - 2R\omega_{bi} S_{xyh}\right) \tag{27}$$

Followed the simple case of [33] the interest is to proposed two classes of design-based calibration estimator in stratified random sampling utilized measure of dispersion such as Gini mean, Dowton, and probability Weighted are all function of the auxiliary information which are insensitive to the presence of outlier or extreme value on data sets [37].

$$\bar{y}_{ARi}=\sum_{h=1}^{l}\Phi_{hi}^{AR}\bar{y}_{hi},i=1,2,3 \tag{28}$$

where Φ_{hi}^{AR} is the new calibration weight such that chi-square function U_5 is defined as

$$\left.\begin{aligned}&U_5=\sum_{h=1}^{l}\left(\Phi_{hi}^{AR}-\Delta_h\right)^2/\Delta_h d_h^*\\&s.t\sum_{h=1}^{l}\Phi_{hi}^{AR}\left(\bar{x}_h+\lambda_{hi(x)}\right)=\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+\lambda_{hi(x)}\right)\end{aligned}\right\},i=1,2,3 \tag{29}$$

$$D_5=\sum_{h=1}^{l}\frac{\left(\Phi_{hi}^{AR}-\Delta_h\right)^2}{\Delta_h d_h^*}-2\delta\left(\sum_{h=1}^{l}\Phi_{hi}^{AR}\left(\bar{x}_h+\lambda_{hi(x)}\right)-\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+\lambda_{hi(x)}\right)\right) \tag{30}$$

where $\lambda_{hi(x)}=G_{MDh}(x)$, $\lambda_{hi(x)}=D_{Mh}(x)$ and $\lambda_{hi(x)}=P_{WMh}(x)$

$$\Phi_{hi}^{AS}=\Delta_h+\Delta_h\sum_{h=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+\lambda_{hi(x)}\right)-\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+\lambda_{hi(x)}\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+\lambda_{hi(x)}\right)\right) \tag{31}$$

$$\left.\begin{aligned}&\bar{y}_{AR1}=\sum_{h=1}^{l}\Delta_h\bar{y}_h\Delta_h\sum_{i=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+G_{MDh}(x)\right)-\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+G_{MDh}(x)\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+G_{MDh}(x)\right)\right)\\&\bar{y}_{AR2}=\sum_{h=1}^{l}\Delta_h\bar{y}_h\Delta_h\sum_{i=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+D_{Mh}(x)\right)-\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+D_{Mh}(x)\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+D_{Mh}(x)\right)\right)\\&\bar{y}_{AR3}=\sum_{h=1}^{l}\Delta_h\bar{y}_h\Delta_h\sum_{i=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+P_{WMh}(x)\right)-\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+P_{WMh}(x)\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h+P_{WMh}(x)\right)\right)\end{aligned}\right\} \tag{32}$$

Another calibration scheme is proposed as

$$\bar{y}_{ASi}=\sum_{h=1}^{l}\Phi_{hi}^{AS}\bar{y}_{hi},i=1,2,3 \tag{33}$$

where Φ_{hi}^{AS} is calibrate weight such that the chi-square distance function U_6 is defined as

minimized
$$\left.\begin{aligned}&U_6=\sum_{h=1}^{l}\left(\Phi_{hi}^{AS}-\Delta_h\right)^2/\Delta_h d_h^*\\&s.t\sum_{h=1}^{l}\Phi_{hi}^{AS}\left(1+\bar{x}_h+\lambda_{hi(x)}\right)=\sum_{h=1}^{l}\Delta_h\left(1+\bar{X}_h+\lambda_{hi(x)}\right)\end{aligned}\right\},i=1,2,3 \tag{34}$$

$$D_6=\sum_{h=1}^{l}\frac{\left(\Phi_{hi}^{AS}-\Delta_h\right)^2}{\Delta_h d_h^*}-2\left(\sum_{h=1}^{l}\Phi_{hi}^{AS}\left(1+\bar{x}_h-\lambda_{hi(x)}\right)-\sum_{h=1}^{l}\Delta_h\left(1+\bar{X}_h-\lambda_{hi(x)}\right)\right) \tag{35}$$

$$\Phi_{hi}^{AS}=\Delta_h+\Delta_h\sum_{h=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+\lambda_{hi(x)}\right)-\sum_{h=1}^{l}\Delta_h\left(1+\bar{X}_h+\lambda_{hi(x)}\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+\lambda_{hi(x)}\right)\right) \tag{36}$$

$$\left.\begin{aligned}&\bar{y}_{AS1}=\sum_{h=1}^{l}\Delta_h\bar{y}_h\Delta_h\sum_{h=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+G_{MDh}(x)\right)-\sum_{h=1}^{l}\Delta_h\left(1+\bar{X}_h+G_{MDh}(x)\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+G_{MDh}(x)\right)\right)\\&\bar{y}_{AS2}=\sum_{h=1}^{l}\Delta_h\bar{y}_h\Delta_h\sum_{h=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+D_{Mh}(x)\right)-\sum_{h=1}^{l}\Delta_h\left(1+\bar{X}_h+D_{Mh}(x)\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+D_{Mh}(x)\right)\right)\\&\bar{y}_{AS3}=\sum_{h=1}^{l}\Delta_h\bar{y}_h\Delta_h\sum_{h=1}^{l}\Delta_h\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+P_{WMh}(x)\right)-\sum_{h=1}^{l}\Delta_h\left(1+\bar{X}_h+P_{WMh}(x)\right)\right)\Big/\left(\sum_{h=1}^{l}\Delta_h\left(1+\bar{x}_h+P_{WMh}(x)\right)\right)\end{aligned}\right\} \tag{37}$$

Bias of the first and second calibrate

$$B\left(\bar{y}_{ARI}\right)=\bar{X}^{-1}\sum_{h=1}^{l}\Delta_h\left(n_h^{-1}-N_h^{-1}\right)\left(R\sigma_{ai}^2S_{xh}^2-\sigma_{ai}S_{xyh}\right) \tag{38}$$

$$B\left(\bar{y}_{ASI}\right)=\bar{X}^{-1}\sum_{h=1}^{l}\Delta_h\left(n_h^{-1}-N_h^{-1}\right)\left(R\sigma_{bi}^2S_{xh}^2-\sigma_{bi}S_{xyh}\right) \tag{39}$$

MSE of the first and second calibrate

$$M\left(\bar{y}_{ARI}\right)=\sum_{h=1}^{l}\Delta_h^2\left(n_h^{-1}-N_h^{-1}\right)\left(S_{yh}^2+R^2\sigma_{ai}^2S_{xh}^2-2\sigma_{ai}S_{xyh}\right) \tag{40}$$

$$M\left(\bar{y}_{ASI}\right)=\sum_{h=1}^{l}\Delta_h^2\left(n_h^{-1}-N_h^{-1}\right)\left(S_{yh}^2+R^2\sigma_{bi}^2S_{xh}^2-2\sigma_{bi}S_{xyh}\right) \tag{41}$$

3. Formulation of Suggested Calibration Estimators

Inspired by the works of [33], this study proposed a novel calibration scheme based on power transformation of existing calibration estimators. Specifically, the work extended the approaches of [33] by applying the nth power to the subject constraints of equations (29) likewise (34), thereby establishing a new class of calibration scheme-type estimators. Furthermore, this study incorporates various robust dispersion measures, including the coefficient of variation (C_{Xh}), Gini Mean Difference (G_{MDh}) ,Downton Method (D_{Mh}) ,Probability Weight Moment (P_{WMh}) , Median of Class (M_{DCh}), Hodge Lehma (H_{LMh}), First-Quarter (F_{Qh}), First-Decile (F_{Dh}) Second-Decile (S_{Dh}),Third-Decile (T_{Dh}).

These measures are calculated from each stratum of auxiliary information x and are designed to be resilient to th presence of outliers or extreme values in the data $x\in\Re^{\oplus}$ With unit $X_{hi}, i=1,2,3,\ldots,N_h$, the

$$\left.\begin{array}{c}
G_{MDh}(x)=2N_h^{-1}\left(N_h-1\right)^{-1}\sum_{i=1}^{N_h}\left(2i-N_h-1\right)X_{hi}\\
D_M(x)=2\sqrt{\pi}N_h^{-1}\left(N_h-1\right)^{-1}\sum_{i=1}^{N}\left(i-(N_h+1)/2\right)X_{hi}\\
P_{WM}(x)=\sqrt{\pi}N_h^{-2}\sum_{i=1}^{N}\left(i-(N_h+1)/2\right)X_{hi}\\
H_{LMh}=\left(Median\left(X_{hj}+X_{hk}\right)/2, 1\le j,k\le N_h\right)
\end{array}\right\} \tag{42}$$

3.1. Proposed First Calibration Scheme

Consider an estimator defined in Eq (1) under stratified random sampling with ch-square distance function given Eq (29)

$$\bar{y}_{T_j}=\sum_{h=1}^{l}\Theta_{1hj}^{SA}\bar{y}_{hj}\forall, j=1,2,3,\ldots,10 \tag{43}$$

C_{Xh}, G_{MDh}, D_{Mh} and P_{WMh} as mention above which all are some conventional measure of dispersion.

where Θ_{1hj}^{SA} and Δ_h new and selection or design calibration weights chi-square distance function χ^2 is

defined as Minimized
$$\left.\begin{array}{l}
\chi^2=\sum_{h=1}^{l}\left(\Theta_{1hj}^{SA}-\Delta_h\right)^2/\Delta_h d_h^*\\
st\sum_{h=1}^{l}\Theta_{1hj}^{SA}\left(\bar{x}_h+\hbar_{hj}(x)\right)^{3/2}=\sum_{h=1}^{l}\Delta_h\left(\bar{X}_h+\hbar_{hj}(x)\right)^{3/2}
\end{array}\right\}\forall j=1,2,\ldots,10 \tag{44}$$

Note that the condition Eq (43) is a requirement given by [18] ignored by all the follower of Sir A.R. Fisher.

Obviously to compute the new weight in conjunction with design weight, Langrage function of Eq [45] is constructed using chi-square subject to problem constraint.

$$D_{\hat{T}_1} = \sum_{i=1}^{l} \frac{\left(\Theta_{1hj}^{SA} - \Delta_h\right)^2}{\Delta_h d_h^*} - 2\eta\left(\sum_{h=1}^{l} \Theta_{1hj}^{SA}\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2} - \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \lambdabar_{hj}(x)\right)^{3/2}\right) \tag{45}$$

where η and d_h^* denotes the Lagrange multiplier and suitable chosen weight which result in different form of the estimators respectively. Partially differentiate eq (45) with respect to Θ_{1hj}^{SA} and η and equating the result to zero.

$$\Theta_{1hj}^{SA} = \Delta_h + \eta d_h^* \Delta_h \left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2} \tag{46}$$

$$\sum_{h=1}^{l} \Theta_{1hj}^{SA}\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2} - \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2} = 0 \tag{47}$$

Solving Eq. (46) and Eq. (47) simultaneously, to get Lagrange multiplier expression for η.

$$\eta = \frac{\left(\sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2} - \sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)\right)^{3/2}}{\sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3}\left(\bar{x}_h + \hbar_{hj}(x)\right)^{-3/2}} \tag{48}$$

Sub the equation (48) in equation (46) for η to obtain selection or design calibrate weight by taken $d^* = \sum_{h=1}^{l} \Theta_{1hj}^{SA}\left(\bar{x}_h + \hbar_{hj}(x)\right)^{-1}$ the new calibrate weight Θ_{1hj}^{SA} is obtain as.

$$\Theta_{1hj}^{SA} = \Delta_h + \Delta_h\left(\sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2} - \sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)\left(\sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1} \tag{49}$$

$$\bar{y}_{T_j} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2} \Big/ \left(\sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right) \tag{50}$$

Remark 1: Note that $\hbar_{hj}(x)$ ten different estimators scheme layout are formulate from each strata

Table 1. members of first proposed calibrate scheme-type for different values of $\hbar_{hj}(x)$ are given

No	Members Proposed Calibrate Weigh	Constants
		$\hbar_{hj}(x)$
1	$\bar{y}_{T_j} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$C_{Xh}\left(N_h \times n_h\right)^{-1}$
2	$\bar{y}_{T_j} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \sum_{h=1}^{l} \Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l} \Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$G_{MDh}\left(N_h \times n_h\right)^{-1}$

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3	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$D_{Mh}(N_h \times n_h)^{-1}$
4	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$P_{WMh}(N_h \times n_h)^{-1}$
5	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$M_{Dh}(N_h \times n_h)^{-1}$
6	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$H_{LMh}(N_h \times n_h)^{-1}$
7	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$F_{Qh}(N_h \times n_h)^{-1}$
8	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$F_{Dh}(N_h \times n_h)^{-1}$
9	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$S_{Dh}(N_h \times n_h)^{-1}$
10	$\bar{y}_{T_j} = \sum_{h=1}^{l}\Delta_h \bar{y}_h \sum_{h=1}^{l}\Delta_h\left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2}\left(\sum_{h=1}^{l}\Delta_h\left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2}\right)^{-1}$	$T_{Dh}(N_h \times n_h)^{-1}$

3.1.1. Properties of the proposed estimators

The MSE of the suggested estimator $\bar{y}_{T_j}$ up first order of approximation by New Tailors series introduced [53] (as cited by ([52],[54])) defined as follow:

$$MSE\left(\bar{y}_{T_j}\right) = \Delta_h \Sigma \Delta_h^T \tag{51}$$

$$\Delta_h = \left[\frac{\partial \bar{y}_{T_j}}{\partial \bar{y}_h}, \frac{\partial \bar{y}_{T_j}}{\partial \bar{x}_h}\right], \bar{y}_h = \bar{Y}_h, \bar{x}_h = \bar{X}_h \tag{52}$$

$$\Sigma = \begin{bmatrix} Var(\bar{y}_h) & Cov(\bar{y}_h, \bar{x}_h) \\ Cov(\bar{y}_h, \bar{x}_h) & Var(\bar{x}_h) \end{bmatrix} \tag{53}$$

where
$$\left.\begin{array}{l} Var\left(\bar{x}_{st}\right) = \sum_{h=1}^{l}\Delta_h^2\left(n_h^{-1} - N_h^{-1}\right)S_{xh}^2 \\ \operatorname{cov}\left(\bar{y}_{st}\bar{x}_{st}\right) = \sum_{h=1}^{l}\Delta_h^2\left(n_h^{-1} - N_h^{-1}\right)S_{xyh}^2 \end{array}\right\} \tag{54}$$

Differentiate $\bar{y}_{T_j}$ for $j = 1,2,\ldots,10$ eq(50) partially with respect two sample mean of each strata that is $\bar{y}_h$ and $\bar{x}_h$ and obtaining the following

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$$\partial \bar{y}_{T_j} \Big/ \partial \bar{y}_h = \sum_{h=1}^{l} \Delta_h \left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2} \Big/ \sum_{h=1}^{l} \Delta_h \left(\bar{x}_h + \hbar_{hj}(x)\right)^{3/2} = 1 \tag{55}$$

$$\partial \bar{y}_{T_j} \Big/ \bar{x}_h = -\frac{3}{2} \sum_{h=1}^{l} \underline{\Delta_h y}_h \sum_{h=1}^{l} \Delta_h \left(\bar{X}_h + \hbar_{hj}(x)\right)^{3/2} \Big/ \sum_{h=1}^{l} \Delta_h \left(\bar{x}_h + \hbar_{hj}(x)\right)^{5/2} = -\frac{3}{2} \sum_{h=1}^{l} \underline{\Delta_h y}_h \Big/ \sum_{h=1}^{l} \Delta_h \left(\bar{X}_h + \hbar_{hj}(x)\right) \tag{56}$$

Recall the expression of equation (51), let define $\partial T_{\overline{yJ}} = -3\bar{Y}\vartheta_{\alpha j}/2$

where $\vartheta_{\alpha j} = \sum_{h=1}^{l} \Delta_h \left(\bar{X}_h + \hbar_{hj}(x)\right)$ the known functions.

$$MSE\left(\bar{y}_{T_j}\right) = \begin{pmatrix} 1 & -3\bar{Y}\vartheta_{\alpha j}/2 \end{pmatrix} \begin{pmatrix} Var(\bar{y}_h) & Cov(\bar{y}_h, \bar{x}_h) \\ Cov(\bar{y}_h, \bar{x}_h) & Var(\bar{x}_h) \end{pmatrix} \begin{pmatrix} 1 \\ -3\bar{Y}\vartheta_{\alpha j}/2 \end{pmatrix} \tag{57}$$

$$MSE\left(\bar{y}_{T_j}\right) = Var(\bar{y}_h) - 3R\vartheta_{\alpha j} Cov(\bar{y}_h, \bar{x}_h) + 2.25R^2 \vartheta_{\alpha j}^2 Var(\bar{x}_h) \tag{58}$$

3.2. Proposed Second Calibration Scheme

Consider an estimator defined in Eq (1) under stratified random sampling with ch-square distance function given eq (34)

$$\bar{y}_{\Gamma_k} = \sum_{h=1}^{l} \Theta_{2hk}^{HA} \bar{y}_{hk} \forall, k = 1,2,\ldots,10 \tag{59}$$

C_{Xh}, G_{MDh}, D_{Mh} and P_{WMh} as mention above which all are some conventional measure of dispersion.

where Θ_{2hk}^{HA} and Δ_h new and selection or design calibration weights chi-square distance function χ^2 is defined as

$$\text{Minimized} \quad \left.\begin{aligned} &\chi^2 = \sum_{h=1}^{l} \left(\Theta_{2hk}^{HA} - \Delta_h\right)^2 \Big/ \Delta_h d_h^* \\ &st \sum_{h=1}^{l} \Theta_{2hk}^{HA} \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{3/2} = \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right)^{3/2} \end{aligned}\right\}, \forall k = 1,2,\ldots,10 \tag{60}$$

Note that the condition [33] is a requirement given by [18] ignored by all the follower of Sir A.R. Fisher. Obviously to compute the new weight in conjunction with design weight, the formulated Langrage function using chi-square distance subject to problem constraint is below:

$$D_{\hat{T}_2} = \sum_{i=1}^{l} \frac{\left(\Theta_{2hk}^{HA} - \Delta_h\right)^2}{\Delta_h d_h^*} - 2\eta\left(\sum_{h=1}^{l} \Theta_{2hk}^{HA} \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{3/2} - \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \lambda\!\!\!^{-}_{hk}(x)\right)^{3/2}\right) \tag{61}$$

Using similar approach in section (45), the following are obtained

Solve for Θ_{2hk}^{HA} by introducing Lagrange Multiplier method and substituted $d^* = \sum_{h=1}^{l} \Theta_{1hj}^{SA} \left(1 + \bar{x}_h + \hbar_{hj}(x)\right)^{-1}$, the new calibrate weight for second scheme in Eq (62) and by putting Eq (62) into Eq (59) the result is as follow respectively.

$$\Theta_{2hk}^{HA} = \Delta_h + \Delta_h \left(\sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right)^{3/2} - \sum_{h=1}^{l} \Delta_h \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{3/2}\right) \left(\sum_{h=1}^{l} \Delta_h \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{3/2}\right)^{-1} \tag{62}$$

$$\bar{y}_{\Gamma_k} = \sum_{h=1}^{l} \Delta_h \bar{y}_h \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right)^{3/2} \Big/ \left(\sum_{h=1}^{l} \Delta_h \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{3/2} \right) \tag{63}$$

Remark 2: Note that from quantity $\hbar_{hk}(x)$ ten different estimators layout can be formulated from each strata similar to Table 1: which stand as the members of second proposed calibrate estimators for different values of $\hbar_{hk}(x)$ such as coefficient of variation (C_{Xh}), Gini Mean Difference (G_{MDh}) ,Downton Method (D_{Mh}) .

3.2.1. Properties of the proposed estimators

Differentiate $\bar{y}_{\Gamma_k}$ for $k = 1,2,\ldots,10$ Eq (63) partially with respect two sample mean of each strata that is $\bar{y}_h$ and $\bar{x}_h$ and obtaining the following

$$\partial \bar{y}_{\Gamma_k} \Big/ \partial \bar{y}_h = \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right)^{3/2} \Big/ \sum_{h=1}^{l} \Delta_h \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{3/2} = 1 \tag{64}$$

$$\partial \bar{y}_{\Gamma_k} \Big/ \bar{x}_h = -\frac{3}{2} \sum_{h=1}^{l} \underline{\Delta_h y}_h \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right)^{3/2} \Big/ \sum_{h=1}^{l} \Delta_h \left(1 + \bar{x}_h + \hbar_{hk}(x)\right)^{5/2} = -\frac{3}{2} \sum_{h=1}^{l} \underline{\Delta_h y}_h \Big/ \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right) \tag{65}$$

Likewise from (51) the mean square error of second calibrate is obtained below Note that $\partial \bar{y}_{\Gamma_k} \big/ \bar{x}_h = -3\bar{Y}\vartheta_{\beta k} \big/ 2$ where $\vartheta_{bk} = \sum_{h=1}^{l} \Delta_h \left(1 + \bar{X}_h + \hbar_{hk}(x)\right)$ the known functions.

$$MSE\left(\bar{y}_{T_j}\right) = Var(\bar{y}_h) - 3R\vartheta_{\beta k} Cov(\bar{y}_h, \bar{x}_h) + 2.25 R^2 \vartheta_{\beta k}^2 Var(\bar{x}_h) \tag{66}$$

4. Empirical Comparison

4.1. Simulation Study

To assess the efficiency of the proposed calibration estimators, a comprehensive simulation study was conducted. The performance of these estimators relative to unbiased ratio estimators suggested by ([33], [38]), as well as the conventional estimators by ([41], [37]) utilized two datasets from Log-normal and Weibull distribution, each consisting of 1,000 units, which were stratified into three distinct groups of sizes 500, 200, and 300, respectively. The characteristics regarding the populations are summarized in Table 2. Using the Simple Random Sampling without Replacement (SRSWOR) method, 10,000 times and the samples of sizes 60, 40, and 50 were drawn from each stratum. The bias, MSE and percentages relative efficiency (PRE) of the proposed and considered estimators were calculated using equations (67), (68) and (69), respectively.

$$Bias(T) = \frac{1}{10000} \sum_{s=1}^{10000} \left(T - \bar{Y}\right) \tag{67}$$

$$MSE(T) = \frac{1}{10000} \sum_{s=1}^{10000} \left(T - \bar{Y}\right)^2 \tag{68}$$

$$PRE(T)=\left(\frac{MSE(t_{ai})}{MSE(t^{*}_{pi})}\right)\times 100 \tag{69}$$

Table 2. The table shown distributions and parameters used

Population	Auxiliary Variable X	Study Variable Y
I	$x_h \sim Lognormal(N_h, \varsigma_h), h=(0.2),(0.3),(0.1)$	$y_h = \beta x_h + x_h^2 + x_h^3 + e_h$
II	$x_h \sim Weibull(N_h, \mu_h, \sigma_h), h=(2,1),(3,1),(4,1)$	

Table 3. Bias, MSE and PRE-Under lognormal distribution

Estimator	Constants Value of Regression (β)								
	0.5			1.0			1.5		
	BIAS	MSE	PRE	BIAS	MSE	PRE	BIAS	MSE	PRE
$\overline{y}_{st}$	6.324142	40958.62	100	6.331417	41047.86	100	6.338692	41137.29	100
[33]									
$\overline{y}_{S}$	-88.9297	40030.8	102.3178	-88.92971	40030.8	102.5407	-88.9297	40030.8	102.7641
[38]									
$\overline{y}_{R1}$	-27.9323	29538.57	138.6615	-27.93238	29538.57	138.9636	-27.9323	29538.57	139.2663
$\overline{y}_{R2}$	-10.1751	28222.94	145.1253	-9.928488	28207.16	145.5229	-9.68185	28191.43	145.9213
[41]									
$\overline{y}_{CE}$	-233.013	24883.62	164.6007	-234.8412	24970.8	164.3835	-236.668	25058.66	164.1639
[37]									
$\overline{y}_{AR1}$	-18.7996	32691.17	125.2896	-18.81747	32715.64	125.4686	-18.8353	32740.14	125.6479
$\overline{y}_{AR2}$	-13.5759	34422.96	118.9864	-13.59504	34460.95	119.1141	-13.6141	34498.99	119.242
$\overline{y}_{AR3}$	-13.5759	34422.96	118.9864	-13.59504	34460.95	119.1141	-13.6141	34498.99	119.242
$\overline{y}_{AS1}$	-13.4382	34468.3	118.8299	-13.45726	34506.64	118.9564	-13.4763	34545.04	119.0831
$\overline{y}_{AS2}$	-10.0308	35586.59	115.0957	-10.04823	35633.68	115.194	-10.0656	35680.85	115.2924
$\overline{y}_{AS3}$	-10.0308	35586.59	115.0957	-10.04823	35633.68	115.194	-10.0656	35680.85	115.2924
Proposed First Calibration Scheme									
$\overline{y}_{T_{01}}$	-118.787	17117.65	239.2772	-118.965	17070.67	240.4585	-119.142	17024.5	241.6359
$\overline{y}_{T_{02}}$	-118.788	17117.57	239.2782	-118.9656	17070.59	240.4595	-119.142	17024.42	241.6369
$\overline{y}_{T_{03}}$	-118.785	17117.91	239.2735	-118.9626	17070.93	240.4548	-119.139	17024.76	241.6321
$\overline{y}_{T_{04}}$	-118.785	17117.91	239.2735	-118.9626	17070.93	240.4548	-119.139	17024.76	241.6321
$\overline{y}_{T_{05}}$	-118.789	17117.34	239.2815	-118.9666	17070.36	240.4629	-119.143	17024.19	241.6402
$\overline{y}_{T_{06}}$	-118.786	17117.36	239.2812	-118.964	17070.37	240.4626	-119.141	17024.2	241.64
$\overline{y}_{T_{07}}$	-118.789	17117.42	239.2804	-118.9668	17070.44	240.4617	-119.144	17024.27	241.6391
$\overline{y}_{T_{08}}$	-118.790	17117.24	239.2829	-118.9681	17070.25	240.4643	-119.145	17024.09	241.6417
$\overline{y}_{T_{09}}$	-118.790	17117.26	239.2826	-118.9678	17070.28	240.464	-119.145	17024.11	241.6414
$\overline{y}_{T_{10}}$	-118.790	17117.29	239.2822	-118.9675	17070.3	240.4636	-119.144	17024.13	241.641

Proposed Second Calibration Scheme									
$\bar{y}_{\Gamma_{01}}$	-81.1169	21070.39	194.3895	-81.26917	21025.99	195.2244	-81.4214	20981.8	196.0618
$\bar{y}_{\Gamma_{02}}$	-81.1172	21070.32	194.3901	-81.26952	21025.93	195.225	-81.4217	20981.73	196.0624
$\bar{y}_{\Gamma_{03}}$	-81.1157	21070.61	194.3875	-81.26801	21026.21	195.2224	-81.4202	20982.02	196.0597
$\bar{y}_{\Gamma_{04}}$	-81.1157	21070.61	194.3875	-81.26801	21026.21	195.2224	-81.4202	20982.02	196.0597
$\bar{y}_{\Gamma_{05}}$	-81.1178	21070.16	194.3916	-81.27005	21025.76	195.2266	-81.4222	20981.56	196.064
$\bar{y}_{\Gamma_{06}}$	-81.1178	21070.16	194.3916	-81.27005	21025.76	195.2266	-81.4222	20981.56	196.064
$\bar{y}_{\Gamma_{07}}$	-81.1179	21070.2	194.3913	-81.27016	21025.8	195.2262	-81.4224	20981.6	196.0636
$\bar{y}_{\Gamma_{08}}$	-81.1185	21070.05	194.3926	-81.27083	21025.66	195.2275	-81.4230	20981.46	196.0649
$\bar{y}_{\Gamma_{09}}$	-81.1184	21070.08	194.3924	-81.27066	21025.68	195.2273	-81.4229	20981.48	196.0647
$\bar{y}_{\Gamma_{10}}$	-81.1182	21070.1	194.3922	-81.27049	21025.7	195.2271	-81.4227	20981.51	196.0645

Table 4. Bias, MSE and PRE-Under Weibull distribution

Estimators	**Constants Value of Regression (β)**								
		0 .5			1.0			1.5	
	BIAS	MSE	PRE	BIAS	MSE	PRE	BIAS	MSE	PRE
$\bar{y}_{st}$	0.032861	1.013222	100	0.034405	1.073366	100	0.035949	1.138374	100
					[33]				
$\bar{y}_{S}$	-0.993380	3.270021	30.98517	-0.99338	3.270021	32.82442	-0.99338	3.270021	34.81243
					[38]				
$\bar{y}_{R1}$	-0.009741	0.761363	133.0798	-0.00974	0.7613637	140.9793	-0097411	0.761363	149.5177
$\bar{y}_{R2}$	0.022137	0.825123	122.7963	0.026969	0.8439541	127.1829	0.031801	0.865194	131.5743
					[41]				
$\bar{y}_{CE}$	-9.552008	9.4652	10.7047	-11.2329	12.97107	8.275071	-12.9139	17.04316	6.679361
					[37]				
$\bar{y}_{AR1}$	-0.006433	0.796614	127.191	-0.006986	0.8034259	133.5986	-0.00754	0.810415	140.468
$\bar{y}_{AR2}$	0.003748	0.866391	116.9473	0.003148	0.8888797	120.7549	0.002548	0.912583	124.7419
$\bar{y}_{AR3}$	0.003748	0.866391	116.9473	0.003148	0.8888797	120.7549	0.002548	0.912583	124.7419
$\bar{y}_{AS1}$	0.007217	0.886262	114.3253	0.006769	0.9135575	117.4929	0.006321	0.942472	120.7859
$\bar{y}_{AS2}$	0.012446	0.914444	110.8019	0.012303	0.9487461	113.1352	0.012160	0.985299	115.5359
$\bar{y}_{AS3}$	0.012446	0.914444	110.8019	0.012303	0.9487461	113.1352	0.012160	0.985299	115.5359
Proposed First Calibration Scheme									
$\bar{y}_{T_{01}}$	-0.129354	0.733879	138.0639	-0.138731	0.7920692	135.5141	-0.14810	0.869157	130.9744
$\bar{y}_{T_{02}}$	-0.129354	0.733885	138.0627	-0.138731	0.7920804	135.5122	-0.14810	0.869174	130.9719
$\bar{y}_{T_{03}}$	-0.129357	0.733885	138.0683	-0.138736	0.7920307	135.5207	-0.14811	0.869100	130.983
$\bar{y}_{T_{04}}$	-0.129357	0.733885	138.0683	-0.138736	0.7920307	135.5207	-0.14811	0.869100	130.983
$\bar{y}_{T_{05}}$	-0.129351	0.733878	138.0641	-0.138728	0.7920685	135.5142	-0.14810	0.869156	130.9745
$\bar{y}_{T_{06}}$	-0.129412	0.733875	138.0644	-0.138800	0.7920666	135.5146	-0.14818	0.869155	130.9748
$\bar{y}_{T_{07}}$	-0.129358	0.733863	138.0668	-0.138736	0.792044	135.5184	-0.14811	0.869120	130.98
$\bar{y}_{T_{08}}$	-0.129354	0.733883	138.0631	-0.138731	0.792077	135.5128	-0.14810	0.869169	130.9726
$\bar{y}_{T_{09}}$	-0.129353	0.733881	138.0634	-0.138730	0.7920745	135.5132	-0.14810	0.869165	130.9732
$\bar{y}_{T_{10}}$	-0.129352	0.733880	138.0636	-0.138729	0.7920723	135.5136	-0.14810	0.869162	130.9736

Proposed Second Calibration Scheme									
$\bar{y}_{\Gamma_{01}}$	-0.057898	0.703632	143.9986	-0.06396	0.6965879	154.089	-0.07003	0.690349	164.8982
$\bar{y}_{\Gamma_{02}}$	-0.057899	0.703631	143.9989	-0.063967	0.6965865	154.0893	-0.07003	0.690348	164.8985
$\bar{y}_{\Gamma_{03}}$	-0.057897	0.703637	143.9976	-0.063965	0.6965931	154.0879	-0.07003	0.690354	164.8969
$\bar{y}_{\Gamma_{04}}$	-0.057897	0.703637	143.9976	-0.063965	0.6965931	154.0879	-0.07003	0.690354	164.8969
$\bar{y}_{\Gamma_{05}}$	-0.057898	0.703632	143.9987	-0.063966	0.6965876	154.0891	-0.07003	0.690349	164.8982
$\bar{y}_{\Gamma_{06}}$	-0.057898	0.703632	143.9987	-0.063966	0.6965876	154.0893	-0.07003	0.690349	164.8982
$\bar{y}_{\Gamma_{07}}$	-0.057897	0.703636	143.9979	-0.063965	0.6965914	154.0883	-0.07003	0.690353	164.8973
$\bar{y}_{\Gamma_{08}}$	-0.057898	0.703632	143.9988	-0.063966	0.6965869	154.0891	-0.07003	0.690348	164.8984
$\bar{y}_{\Gamma_{09}}$	-0.057898	0.703632	143.9988	-0.063966	0.6965871	154.0892	-0.07003	0.690348	164.8984
$\bar{y}_{\Gamma_{10}}$	-0.057898	0.703632	143.9987	-0.063966	0.6965873	154.0892	-0.07003	0.690349	164.8983

5. Discussion

Tables 3 and 4 showed the metric (Bias, MSE, PRE) of the performance of proposed calibrate together with existing estimators consider on study using population (LogNormal) and (Weibull) respectively as showed in Table 2 for different value of regression β = (0.5,1.0,1.5). The results revealed in Tables 3 and 4 under (coefficients of non-linear regression response character model) the proposed calibrate possess the highest percentage relative efficiency (PRE) compare to usual ratio and conventional ratio without non exception. The proposed calibrate scheme type efficient rows is continue from Tables 3 and 4 at different ranges of coefficient of non-linear regression model. After conducting a detailed numerical study, the following key findings are identified.

i. The proposed calibration schemes generally exhibit low bias and mean square error (MSE) values, indicating more efficiency performance of the proposed calibration estimators.
ii. The Tables 3 and 4 demonstrate precision (PRE) values are generally high without any exception, indicating that the proposed estimators are more efficient than the benchmark estimator.
iii. The performance of the proposed calibration schemes varies slightly high across different values of the regression parameter model of study variable or as increase in a coefficient of non-linear regression, the efficiency of the all estimators increases except the Weibull distribution which exhibit revers from first calibrate scheme but still maintained most efficiency evident from Tables 3 and 4.

6. Conclusion

The proposed calibration estimators consistently outperform more than its counterparts existing calibration estimators review in this study within specific parameter ranges, showcasing the robustness of the proposed calibration estimators. The Percentage Relative Efficiency (PRE) of the proposed calibrate estimator is superior. The proposed calibration scheme PRE is consistent defect the changes distribution and change of coefficient of parameter of the non-regression of study variable.

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